Viscous Heating in Laminar Slit Flow.

P.1 From Eq. A of Table 10.2-3 we may write:

$$0 = k\frac{d^2T}{dx^2} + \mu\left(\frac{dv_z}{dx}\right)^2$$

It may readily be shown that the velocity profile is:

$$v_z = v_{max}\left[1 - (x/B)^2\right]$$

and therefore that

$$(dv_z/dx)^2 = v_{max}^2\,(4x^2/B^4)$$

The energy equation is then:

$$(d^2T/dx^2) = -4\mu v_{max}^2 x^2/kB^4$$

The boundary conditions are:

At $x = B$, $T = T_0$
At $x = -B$, $T = T_0$

Solving the energy equation with these boundary conditions we find:

$$T - T_0 = \tfrac{1}{3}(\mu v_{max}^2/k)\left[1 - (x/B)^4\right]$$

Heat Conduction from a Sphere to a Stagnent Fluid.

P.2 a. The differential equation is the same as that derived in Prob. 9.F. For constant thermal conductivity this becomes:

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

b. Integration twice with respect to r gives:

$$T = -\frac{C_1}{r} + C_2$$

The boundary conditions are:

At $r = R$ $\quad T = T_R$

At $r = \infty$ $\quad T = T_\infty$

Therefore the temperature distribution is:

$$\frac{T - T_\infty}{T_R - T_\infty} = \frac{R}{r}$$

c. The heat flux at the wall is then:

$$q_r\Big|_{r=R} = -k\frac{dT}{dr}\Big|_{r=R} = +k\frac{(T_R - T_\infty)}{R}$$

We now compare this expression with Newton's law of cooling:

$$q_r\Big|_{r=R} = h\,(T_R - T_\infty)$$

which defines h. Equating these last two expressions we get:

$h = k/R$

Question # 3

$A = 900\ cm^2.$

$\Delta Y = 5 \times 10^{-1} cm.$

$Q = 3.0\ W.$ or $3.0\ J/s$

$\Delta T = 26 - 24 = 2\ K.$

$$\frac{Q}{A} = -k \frac{dT}{dx}$$

$$k = \frac{Q \Delta y}{A \Delta T} \qquad \frac{Q}{A} = -k \left(\frac{\Delta T}{\Delta x}\right)$$

$$k = \frac{3.0\ J s^{-1} \times 5\ 10^{-1} cm.}{2K \times 900\ cm^2} = \boxed{8.3\ 10^{-4} J . s^{-1} cm^{-1} K^{-1}}$$

Question # 4

$$q_r = -k \underbrace{A}_{2\pi r L} \frac{dT}{dr}$$

$$q_r \frac{dr}{kr} = -2\pi L\, dT$$

$$q = \frac{2\pi\ L\ (\Delta T)}{\frac{1}{K_p} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{K_g} \ln\left(\frac{r_3}{r_2}\right)}$$

$$\frac{q}{L} = \frac{2\pi\ \Delta T}{\frac{1}{K_p} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{K_g} \ln\left(\frac{r_3}{r_2}\right)} = \underline{\underline{216.6\ Btu/hr.ft}}$$